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NetCam SSSTC

Randomized exploration in environmental and surveillance applications

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Introduction

In a wide variety of applications one wants to find the maxima of a scalar function over a region of interest. We use a discrete time Markov Chain Monte Carlo (MCMC) method to identify the positions of the maxima. The application of this randomized search algorithm is demonstrated on two different problems: an underwater exploration scenario and a surveillance scenario. In the latter we obtain a stochastic patrolling strategy that monitors an area while taking high-value targets into account.

Search Algorithm

Goal: Identify the locations of the maxima of a scalar concentration function $C : \mathcal{X} \to [0, 1]$ on a compact domain $\mathcal{X} \subset \mathbb{R}^2$.

- Search performed by N agents, each with simplified discrete-time dynamics:

$$\begin{pmatrix} x_{k+1} \\ \theta_{k+1} \end{pmatrix} = \begin{pmatrix} x_k + v(\theta_k)T \\ u_k \end{pmatrix} = \begin{pmatrix} x_k + \bar{v} \begin{pmatrix} \cos(\theta_k) \\ \sin(\theta_k) \end{pmatrix} T \\ u_k \end{pmatrix}, \qquad (1)$$

  with position $x \in \mathcal{X}$, heading angle $\theta \in [0, 2\pi]$, constant speed $\bar{v}$, sampling period T and control input u.
- Markovian controller, generating a discrete-time Markov chain $\{s_k\}_{k\geq 0}$ for each agent, where $s_k := (x_k, \theta_k) \in \mathcal{S} := \mathcal{X} \times [0, 2\pi]$.
- The MCMC method to compute the input u_k is outlined in Algorithm 1.

Algorithm 1 MCMC Search Algorithm

Require: Initial state $s_0 = (x_0, \theta_0)$

1: set $k = 0$
2: **loop**
3: update $x_{k+1} = x_k + v(\theta_k)T$
4: generate proposal angle $\tilde{\theta}_{k+1}$
5: calculate the acceptance probability $\alpha(s_k)$
6: update $\theta_{k+1} = u_k$, where

$$u_k = \begin{cases} \tilde{\theta}_{k+1} & \text{w. p. } \alpha(s_k) \\ \theta_k & \text{w. p. } 1 - \alpha(s_k) \end{cases}$$

7: set $k = k + 1$
8: **end loop**

In essence:

- A new heading angle is proposed each step from a distribution q.
- If it is accepted, the agents *turns*, i.e. changes direction in the next step.
- If rejected, the agent keeps going in its previous direction.
- The positions are determined through the dynamics.
- Main design choice are distribution q to generate the proposals and acceptance criterion α.
- Choices for q and α only need to adhere the criteria below.

Design criteria:

- Proposals drawn from q must not lead the agent outside $\mathcal{X}$ in the next step.
- α is monotonically increasing in C, increasing the probability of a *turn* in high-concentration areas. Hence the agents spend more time in these regions.
- α depends continuously on s (technical condition).
- $\alpha = 1$ near the boundary of $\mathcal{X}$. Forces the agents to turn, before leaving $\mathcal{X}$.

Result: Agents will randomly explore the space $\mathcal{X}$. The distribution of the observed agents' positions over the mission time approximates the function C. I.e. the peaks of the distribution coincide with the location of the maxima.

Application 1: Autonomous Underwater Vehicle (AUV)

Underwater exploration scenario to identify the source locations of substances, e.g., fresh water or chemical pollution, with the help of AUVs.

- Case study on circular $\mathcal{X}$ with radius $R = 400$m and field

$$C(x) = \sum_{i=1}^{3} w_i e^{-m\|x - x_{s,i}\|}.$$

- Choice of a sigmoidal function as acceptance criterion:

$$\alpha(s_k) = 1 - e^{-(kC(x_k)^J)}\lambda(s_k), \qquad (2)$$

  with tuned parameter $k = 100$ and $J = 0.1$. λ is a designed *border* function, ensuring that $\alpha = 1$ close to the border.

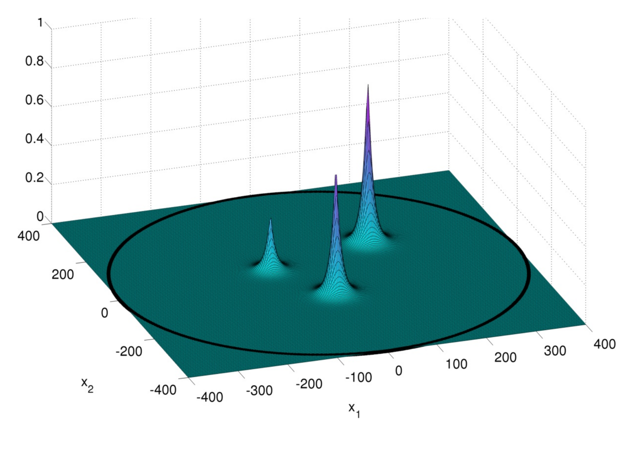
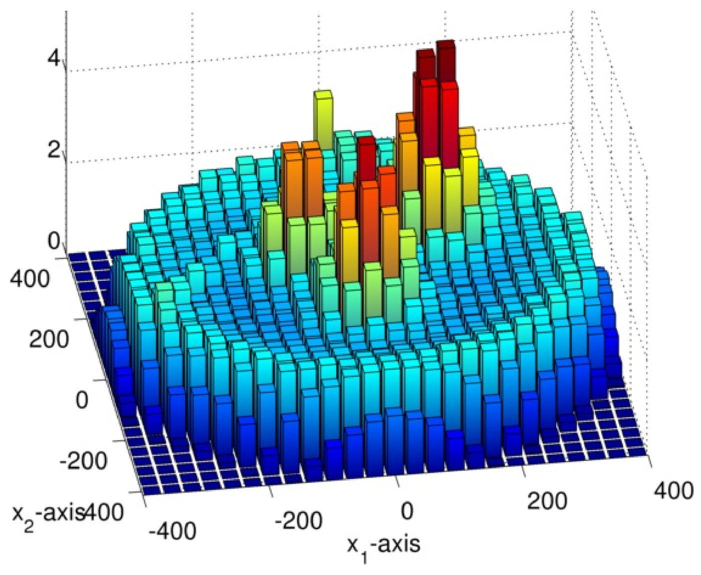


Figure 1: Parameters: $x_{s,1} = (-80, 50)$, $w_1 = 0.3$, $x_{s,2} = (0, -100)$, $w_2 = 0.7$, $x_{s,3} = (140, 120)$, $w_3 = 0.9$ and $m = 0.1$.

Figures 2 and 3 depict the discretized spatial distributions of the AUVs for different number of agents and different mission times.



increasing → number of agents & mission time

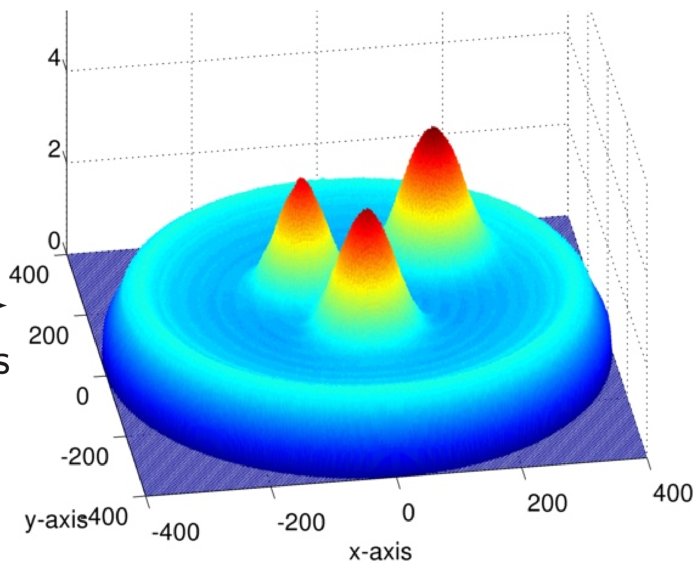


Figure 2: 5 AUVs for 5 hours

Figure 3: 1000 AUVs for 5000 hours

Details and the numerical convergence analysis of the Markov chain for this case study are presented in [1].

Application 2: Camera Surveillance

Surveillance scenario for patrolling an area $\mathcal{G} \subset \mathbb{R}^2$ with N cameras. Extend benefit of stochastic patrolling strategies [2] by qualitative constraints, i.e, higher observation probability of important objects.

Specification phase:

- Desired observation probability of locations $g \in \mathcal{G}$ given as *covering function* $\pi(g)$.
- Aim: Achieve π as best as possible.
- To apply the MCMC search, a function C_n is needed for each camera.
- Synthesis of C_n on the local pan-tilt spaces $\mathcal{X}_n$ by solving the LP (3),

$$\begin{aligned} \min_{\xi_n, c} \quad & \|c\pi(g) - \sum_{n=1}^{N} B_n^\top \xi_n\|_\infty \\ \text{s.t.} \quad & (\xi_n)_j \geq 0 \\ & \sum_j (\xi_n)_j = 1 \\ & \forall n \in \{1, \cdots, N\} \end{aligned} \qquad (3)$$

  with distributions of camera positions ξ_n, matrices $B_n^\top$ mapping pan-tilt configurations on $\mathcal{X}_n$ to observed areas on $\mathcal{G}$ and scaling factor c.
- Obtain $C_n(x) = \sum_{l=1}^{N_X} w_l \mathcal{N}(x|\mu_l, \Sigma)$ s.t. $C_n(X) = \xi_n^*$, (sum of weighted Gaussians), with grid points $X \in \mathcal{X}_n$.

Patrolling phase:

- Case study with 2 cameras and desired π depicted in Figure 4.
- The acceptance criterion was chosen similar to (2) with different tuning.
- Simulated distributions of camera positions (Figures 8 and 9) approximate the designed C_n (Figures 5 and 6). Figure 7 shows resulting covering $\hat{\pi}$.

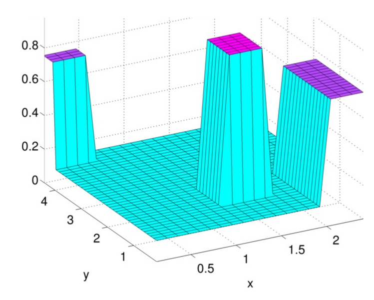


Figure 4: Desired covering π

Specification from $\mathcal{G}$ → to $\mathcal{X}_n$

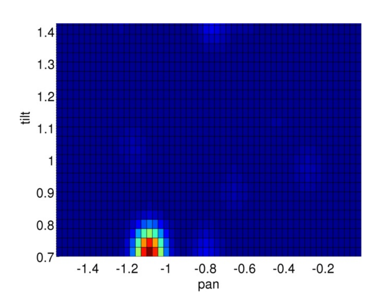


Figure 5: C_1 of camera 1

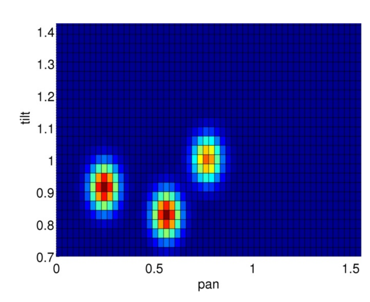


Figure 6: C_2 of camera 2

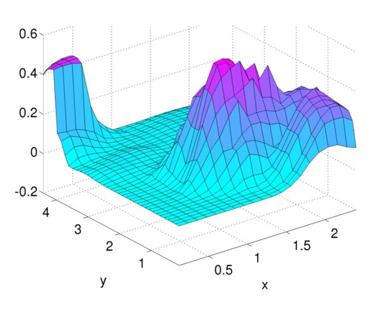


Figure 7: Achieved covering $\hat{\pi}$ (ideal)

Simulation from $\mathcal{X}_n$ ← to $\mathcal{G}$

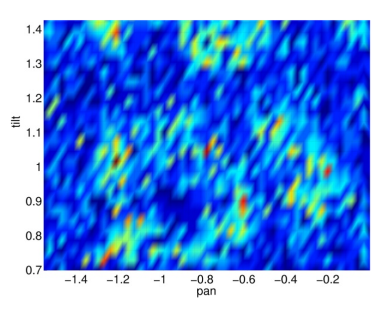


Figure 8: Simulated positions camera 1

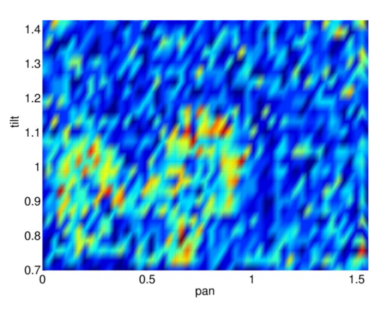


Figure 9: Simulated positions camera 2

Acknowledgement

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[2] D. M. Raimondo and N. Kariotoglou and S. Summers and J. Lygeros: *Probabilistic certification of pan-tilt-zoom camera surveillance systems.* CDC 2010.