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Centralized and Distributed AC OPF Algorithms for Radial Active Distribution Networks

K. Christakou, D.-C. Tomozei, J.-Y. Le Boudec, M. Paolone

Laboratory for Communications and Applications LCA2 ,Distributed Electrical Systems Laboratory DESL

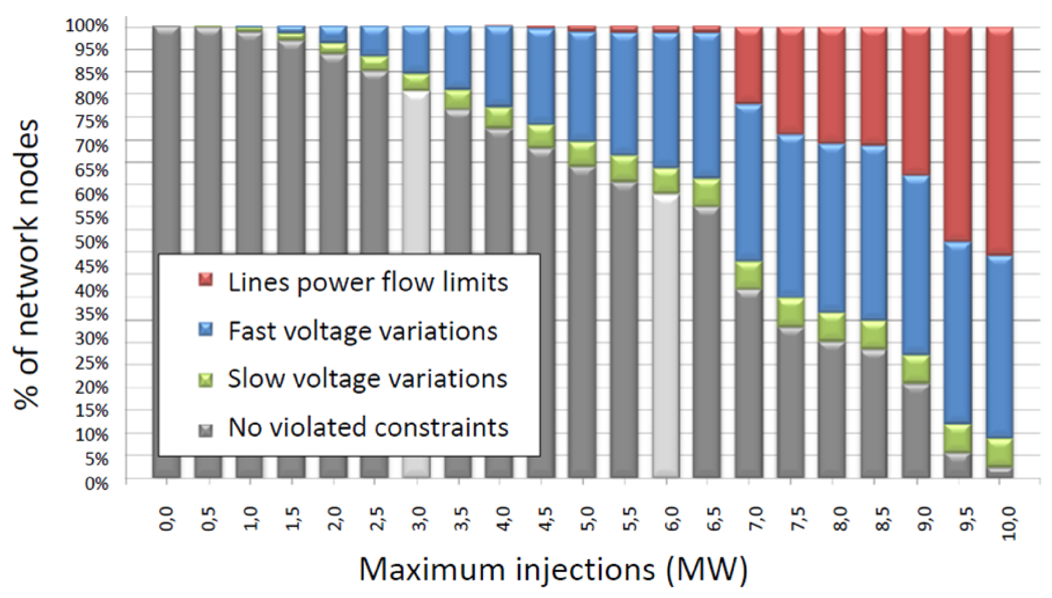
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

The AC OPF problem

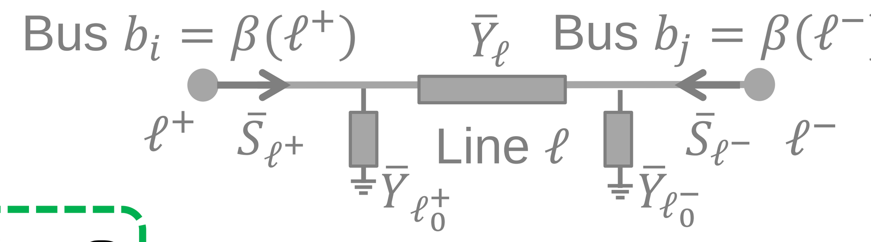
What is the OPF problem?

The OPF computes an optimal operating point for a power system that minimizes an appropriate cost function subject to operational constraints.

Why is the OPF important in ADNs?



Classic formulation: Network with $\mathcal{B}$ buses, $\mathcal{L}$ lines, G generators, C loads



$$\min_{\bar{S}_g,\bar{S}_c,\bar{S}_{\ell^+},\bar{S}_{\ell^-},\bar{I}_{\ell^+},\bar{I}_{\ell^-},\bar{V}_b} \sum_{g\in G} C_g(\bar{S}_g) + \sum_{c\in C} C_c(\bar{S}_c)$$

$$\sum_{g\in b}\bar{S}_g - \sum_{c\in b}\bar{S}_c + \sum_{\beta(\ell^+)=b}\bar{S}_{\ell^+} + \sum_{\beta(\ell^-)=b}\bar{S}_{\ell^-} = 0\ , \forall\, b\in\mathcal{B}$$

1. Bus power balance constraints

$$\bar{S}_{\ell^+} = \bar{V}_{\beta(\ell^+)}\bar{I}_{\ell^+},\ \ \bar{S}_{\ell^-} = \bar{V}_{\beta(\ell^-)}\bar{I}_{\ell^-}, \quad \forall\,\ell\in\mathcal{L}$$
$$\bar{I}_{\ell^+} = \bar{Y}_\ell\left(\bar{V}_{\beta(\ell^+)} - \bar{V}_{\beta(\ell^-)}\right) + \bar{Y}_{\ell_0^+}\bar{V}_{\beta(\ell^+)},\ \forall\,\ell\in\mathcal{L}$$
$$\bar{I}_{\ell^-} = \bar{Y}_\ell\left(\bar{V}_{\beta(\ell^-)} - \bar{V}_{\beta(\ell^+)}\right) + \bar{Y}_{\ell_0^-}\bar{V}_{\beta(\ell^-)},\ \forall\,\ell\in\mathcal{L}$$

2. Power flow equations

Non-convex

$$V_{min} \le |\bar{V}_b| \le V_{max},\ \forall\, b\in\mathcal{B}$$
$$|\bar{I}_{\ell^+}| \le I_{\ell_{max}}, |\bar{I}_{\ell^-}| \le I_{\ell_{max}}, \forall\,\ell\in\mathcal{L}$$

3. Voltage and line-flows operational limits

$$\bar{S}_g \in \mathcal{H}_g, \forall g\in G\ \ and\ \ \bar{S}_c \in \mathcal{H}_c, \forall c\in C$$

4. Generators capabilities and load limits

Problem: Hard non-convex constrained optimization, >> variables
Solution: Design of **provably convergent algorithms**, easily **distributable**

Centralized OPF Algorithm

- **Introduce slack variables and constraints:** $\bar{E}_{\ell^+} = \bar{V}_{\beta(\ell^+)}, \bar{E}_{\ell^-} = \bar{V}_{\beta(\ell^-)}, \forall\,\ell\in\mathcal{L}$
- **Augmented Lagrangian (partial elimination of constraints):**

$$L_\rho = \sum_{g\in G} C_g(\bar{S}_g) + \sum_{c\in C} C_c(\bar{S}_c) + \frac{\rho}{2}\{\sum_{\ell\in\mathcal{L}}\left|\bar{E}_{\ell^+} - \bar{V}_{\beta(\ell^+)} + \bar{\lambda}_{\ell^+}\right|^2 +$$
$$\sum_{\ell\in\mathcal{L}}\left|\bar{E}_{\ell^-} - \bar{V}_{\beta(\ell^-)} + \bar{\lambda}_{\ell^-}\right|^2 + \sum_{b\in B}\left|\sum_{g\in b}\bar{S}_g - \sum_{c\in b}\bar{S}_c + \sum_{\beta(\ell^+)}\bar{S}_{\ell^+} + \sum_{\beta(\ell^-)}\bar{S}_{\ell^-} + \bar{\lambda}_b\right|^2\}$$

- **Iterative solution via method of multipliers :**
  - Initialize κ=0, control variables, Lagrange multipliers, (ρ^κ), $\rho^\kappa \to \infty$
  1: **repeat**
  2: $(\bar{S}_g, \bar{S}_c, \bar{E}_{\ell^+}, \bar{E}_{\ell^-}, \bar{V}_b)^{k+1} \rightarrow$ min L_{ρ^κ} s.t. constraints (3,4) for fixed $\bar{\lambda}^\kappa$ **(P1)**
  3: update Lagrange multipliers $\bar{\lambda}^{\kappa+1} = \bar{\lambda}^\kappa + \nabla_{\bar{\lambda}} L_{\rho^\kappa}\big|_{(\bar{S}_g,\bar{S}_c,\bar{E}_{\ell^+},\bar{E}_{\ell^-},\bar{V}_b)^{k+1}}$
  4: $\kappa \to \kappa+1$
  5: **until** $\kappa = \kappa_{max}$ **or** the change in the Lagrange multipliers is $\le \delta$

Proof of convergence to a local minimum

Distributed OPF Algorithm

If $\bar{E}_{\ell^+}, \bar{E}_{\ell^-}$ are fixed, **(P1)** decouples across buses

- **Primal decomposition on** $\bar{E}_{\ell^+}, \bar{E}_{\ell^-}$
  - Iterative (asynchronous) solution for **(P1)**:

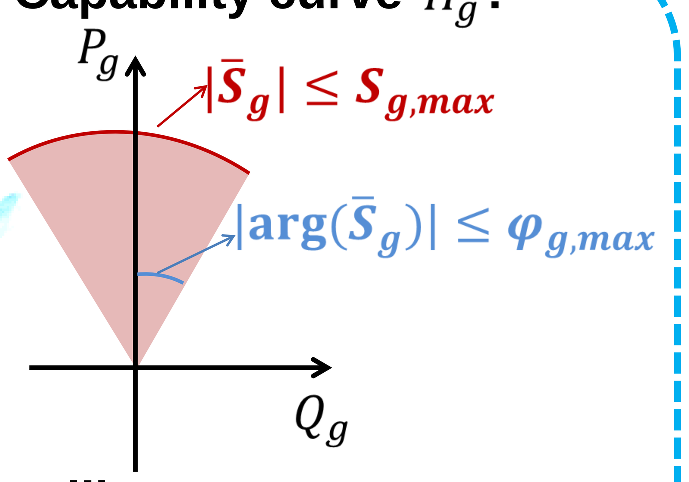
1: **repeat**
2: buses update in parallel $(\bar{S}_c)_{c\in b}, (\bar{S}_g)_{g\in b}, \bar{V}_b$ and send this information to the adjacent lines
3: lines update in parallel the voltages at their two ends by gradient descent step and send the new $\bar{E}_{\ell^+}, \bar{E}_{\ell^-}, \bar{S}_{\ell^+}, \bar{S}_{\ell^-}, \bar{I}_{\ell^+}, \bar{I}_{\ell^-}$ to the incident buses
4: $n \to n+1$
5: **until** $n = n_{max}$ **or** the change in $\bar{E}_{\ell^+}, \bar{E}_{\ell^-}$ is $\le \varepsilon$

Application: Fair Scheduling of Distributed PV Units

Case study:

- IEEE 13-nodes test feeder
- Distributed controllable PV

- **Capability curve** H_g:

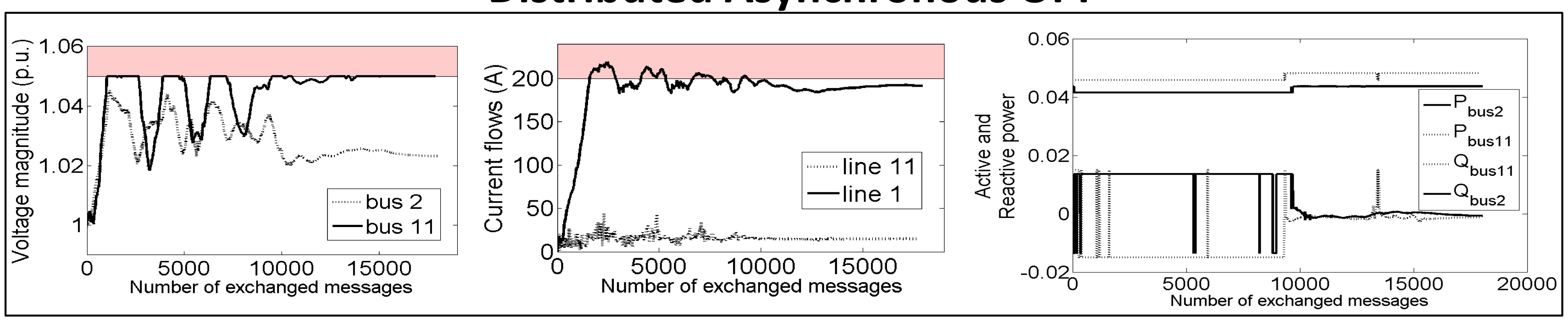


- **Utility:** $U_g = \log(P_g) - \eta({P_g}^2 + {Q_g}^2)$

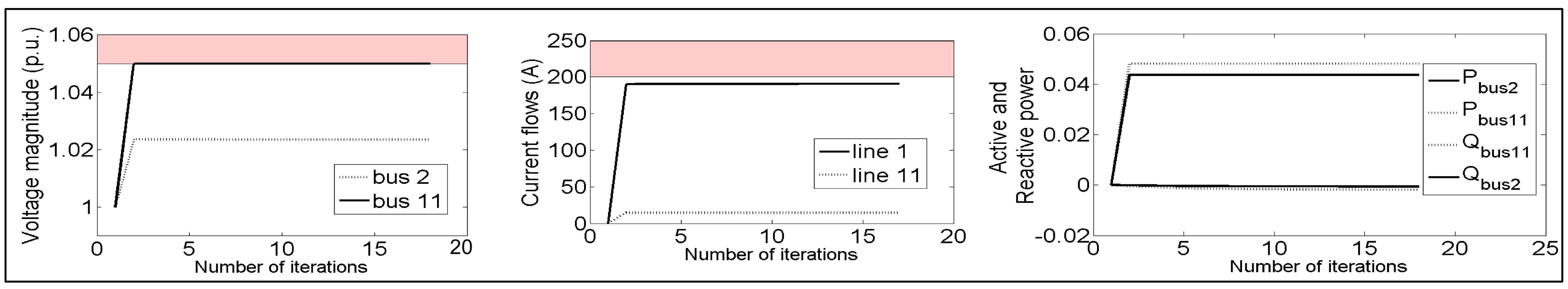
- Non dispatchable demand

- **Capability curve** H_c : $(P_c, Q_c) = (P_{c,0}\,, Q_{c,0})$
- **Utility:** $U_c = 0$

Distributed Asynchronous OPF



Centralized OPF



Reference

1. K. Christakou, D.-C. Tomozei, J.-Y. Le Boudec, and M. Paolone, *AC OPF in Radial Distribution Networks-Parts I, II, submitted to IEEE Transactions to Power Systems, 2015 (arXiv preprint arXiv:1503.06809)*